Indeterminacy Analysis: Model, Method, and Results

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1 Model Environment

The model for this analysis is a slightly modified version of the textbook 3-equation new-Keynesian DSGE model presented by Galí (2008) that connects the output gap (x_t), inflation (π_t), and interest rate (i_t). Here is the model:

Euler Equation:

$$x_t = \frac{1}{1+h}\mathbb{E}_t x_{t+1} + \frac{h}{1+h}x_{t-1} - \frac{1-h}{1+h}\sigma^{-1}(i_t - \mathbb{E}_t\pi_{t+1} - r_t) \tag{1}$$

New-Keynesian Phillips Curve:

$$\pi_t = \frac{\beta}{1+\beta\gamma}\mathbb{E}_t\pi_{t+1} + \frac{\gamma}{1+\beta\gamma}\mathbb{E}_t\pi_{t-1} + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha(1+\beta\gamma)}\left(\varphi x_t + \frac{\sigma}{1-h}(x_t - hx_{t-1})\right) + u_t \tag{2}$$

Taylor Rule:

$$i_t = \rho i_{t-1} + (1-\rho)(\chi_\pi \pi_t + \chi_x x_t) + \varepsilon_t^i \tag{3}$$

AR(1) Processes:

$$r_t = \rho_r r_{t-1} + \varepsilon_t^r \tag{4}$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \tag{5}$$

where $0 < h < 1$ is the degree of external habit formation, $\sigma > 0$ is the intertemporal elasticity of consumption, $\beta = 0.99$ is the discount factor, $0 < \gamma < 1$ is the degree of price indexation, $0 < \alpha < 1$ is the Calvo price stickiness parameter, $\varphi > 0$ is the inverse Frisch elasticity of labor supply, $0 < \rho < 1$ is the degree of interest rate smoothing, χ_π, $\chi_x > 0$ are the monetary policy response to inflation and output gap respectively, and

$0 < \rho_r$, $\rho_u < 1$ are the persistence of the disturbance processes. The model variables are subject to iid and normally distributed exogenous demand (ε_t^r), supply (ε_t^u), and monetary policy (ε_t^i) shocks.

Only the version under determinacy is presented here. For indeterminacy, a sunspot shock (ε_t^ξ) is included in the evolution of inflation expectations. This sunspot shock is correlated with the structural demand, supply, and monetary policy shocks in the economy.[1] The method presented by Bianchi and Nicoló (2021) is used to account for one degree of indeterminacy in the estimation code. Please refer to their paper directly for more information on how indeterminacy is included in the analysis.

2 Empirical Methodology and Results

The model listed above is estimated using data for all three U.S. macro indicators - x_t, π_t, and i_t. All data for these indicators are collected at a quarterly frequency from Q1 2009 to Q4 2022. Output gap is computed as the log difference between current period real GDP and real potential GDP. Inflation is computed as the annualized log difference between the current and previous period of the PCE price index - the Fed's preferred measure of inflation (Yellen, 2015). Data for all output and inflation metrics were collected from the FRED database. Since the ZLB period of monetary policy is in effect for almost the entirety of the sample period, the Wu and Xia (2016) shadow federal funds rate is used to accurately capture the stance of monetary policy. Finally, given the importance of expectations in a determinacy analysis, one year ahead inflation expectations are collected from the Michigan Survey of Consumers and included in the estimation. The observation equations are as follows:

$$OBS_t = \begin{bmatrix} x_t^{obs} \\ \pi_t^{obs} \\ i_t^{obs} \\ \mathbb{E}\pi_t^{obs} \end{bmatrix} = \begin{bmatrix} x^* \\ \pi^* \\ i^* \\ \pi^* \end{bmatrix} + \begin{bmatrix} x_t \\ 4\pi_t \\ 4i_t \\ \mathbb{E}_t \sum_{i=1}^{4} \pi_{t+i} + \varepsilon_t^m \end{bmatrix} \tag{6}$$

where x^*, π^*, and $i*$ are the growth trends of output gap, annualized inflation, and annualized interest rates respectively. ε_t^m is an exogenous measurement error. The model is estimated using a Bayesian MCMC Metropolis-Hastings algorithm using one chain of 500,000 draws with a 40% burn-in.[2] All parameters are identified and trace plots confirm parameter convergence. Priors for the estimated parameters (see Table 1) are chosen as per Smets and Wouters (2007) with the trends adjusted to accurately reflect the post-financial crisis period and shock deviations expanded. The posterior means for all parameters and shocks, as well as the marginal likelihoods, under determinacy and indeterminacy, are presented in Table 1 below.

[1]See Lubik and Schorfheide (2004) for more information on indeterminacy and sunspots.
[2]See An and Schorfheide (2007), Fernández-Villaverde (2010), and Herbst and Schorfheide (2015) for an overview of Bayesian MCMC estimation methods pertaining to DSGE models.

Parameter	Description	Prior	Posterior	
			Determinacy	Indeterminacy
Structural Parameters				
h	Habit formation	$\mathbf{B}(0.70, 0.10)$	0.5286	0.4813
σ	IES	$\mathbf{\Gamma}(1.50, 0.37)$	1.2492	0.4508
α	Calvo factor	$\mathbf{B}(0.50, 0.10)$	0.8697	0.7743
γ	Price Indexation	$\mathbf{B}(0.50, 0.15)$	0.2306	0.2000
φ	Inv. Frisch	$\mathbf{N}(2.00, 0.75)$	1.2326	0.6483
χ_π	Taylor Rule	$\mathbf{N}(1.50, 0.75)$	1.1309	0.5738
χ_x	Taylor Rule	$\mathbf{N}(0.13, 0.06)$	0.1635	0.1988
ρ	Rate smoothing	$\mathbf{B}(0.70, 0.10)$	0.8781	0.8474
x^*	Trend	$\mathbf{N}(0.50, 1.00)$	-1.1906	-0.4729
π^*	Trend	$\mathbf{\Gamma}(2.00, 1.00)$	3.8144	2.4743
i^*	Trend	$\mathbf{\Gamma}(1.00, 0.50)$	1.8161	1.1183
Shock Processes				
Persistence				
ρ_r	Demand	$\mathbf{B}(0.50, 0.20)$	0.5652	0.7699
ρ_u	Supply	$\mathbf{B}(0.50, 0.20)$	0.6443	0.3753
Deviation				
σ_r	Demand	$\mathbf{\Gamma}^{-1}(0.50, 2.00)$	3.0684	0.7780
σ_u	Supply	$\mathbf{\Gamma}^{-1}(0.30, 2.00)$	0.1288	0.2096
σ_i	Monetary Policy	$\mathbf{\Gamma}^{-1}(0.30, 2.00)$	0.1470	0.1427
σ_m	Measurement	$\mathbf{\Gamma}^{-1}(0.50, 2.00)$	0.9511	0.1755
σ_ξ	Sunspot	$\mathbf{\Gamma}^{-1}(0.50, 2.00)$	-	0.4526
Correlation				
$\rho_{\xi,r}$	Sunspot, Demand	$\mathbf{U}(-1.00, 1.00)$	-	0.5155
$\rho_{\xi,u}$	Sunspot, Supply	$\mathbf{U}(-1.00, 1.00)$	-	0.7051
$\rho_{\xi,i}$	Sunspot, Monetary	$\mathbf{U}(-1.00, 1.00)$	-	-0.2711
Marginal Likelihood (Modified Harmonic Mean)			-375.41	-340.01

Table 1: Estimated Posterior Means and Marginal Likelihoods

Note: For the priors, symbols represent distributions in the following manner: $\mathbf{B}$ - Beta, $\mathbf{\Gamma}$ - Gamma, $\mathbf{U}$ - Uniform, $\mathbf{N}$ - Normal, and $\mathbf{\Gamma}^{-1}$ - Inverse Gamma. All prior distributions are presented with means and standard deviations in parentheses except $\mathbf{U}$ which shows lower and upper bounds. 90% credible intervals are available upon request. Marginal likelihoods are computed using Geweke (1999)'s modified harmonic mean approach.

References

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