Fiscal Inflation Analysis: Model, Method, and Results

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1 Model Environment

The model for this analysis inserts hand-to-mouth consumers as introduced in Galí, López-Salido, and Vallés (2007) ('GLV') into the benchmark Smets and Wouters (2007) ('SW') medium-scale, New-Keynesian framework. The equilibrium log-linearized equations are as follows:

$$c_t^o = \frac{h}{1+h}c_{t-1}^o + \frac{1}{1+h}E_t c_{t+1}^o - \frac{1-h}{1+h}(r_t - E_t\pi_{t+1} + \varepsilon_t^b) \tag{1}$$

$$c_t^r = \frac{1-\alpha}{\mu_p^{ss}\gamma_c}(w_t + n_t^r) - \gamma_c^{-1}\tau_t^r \tag{2}$$

$$c_t = \lambda c_t^r + (1-\lambda)c_t^o \tag{3}$$

$$n_t = \lambda n_t^r + (1-\lambda)n_t^o \tag{4}$$

$$w_t = \frac{1}{1+\beta}w_{t-1} + \frac{\beta}{1+\beta}(E_t w_{t+1} + E_t\pi_{t+1}) - \frac{1+\beta\iota_w}{1+\beta}\pi_t + \frac{\iota_w}{1+\beta}\pi_{t-1} - \frac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w(1+\beta)}\mu_t^w + \varepsilon_t^w \tag{5}$$

$$\mu_t^w = w_t - \frac{1}{1-h}(c_t^o - hc_{t-1}^o) - \sigma_l n_t^o \tag{6}$$

$$\mu_t^w = w_t - c_t^r - \sigma_l n_t^r \tag{7}$$

$$i_t = \frac{1}{1+\beta}i_{t-1} + \frac{\beta}{1+\beta}E_t i_{t+1} + \frac{1}{\varphi(1+\beta)}q_t + \varepsilon_t^i \tag{8}$$

$$k_t = (1-\delta)k_{t-1} + \delta i_t + (\delta(1+\beta)\varphi)\varepsilon_t^i \tag{9}$$

$$z_t = \frac{\psi}{1-\psi}r_t^k \tag{10}$$

$$q_t = \beta(1-\delta)E_t q_{t+1} + [1+\beta(1-\delta)]E_t r_{t+1}^k - (r_t - E_t\pi_{t+1} + \varepsilon_t^b) \tag{11}$$

$$\pi_t = \frac{\iota_p}{1+\iota_p\beta}\pi_{t-1} + \frac{\beta}{1+\iota_p\beta}E_t\pi_{t+1} - \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\iota_p\beta)\xi_p}\mu_t^p + \varepsilon_t^p \tag{12}$$

$$\mu_t^p = (y_t - n_t) - w_t \tag{13}$$

$$r_t^k = -(k_t - n_t) + w_t \tag{14}$$

$$y_t = (1-\alpha)n_t + \alpha k_{t-1} + \alpha z_t + \varepsilon_t^a \tag{15}$$

$$y_t = \gamma_c c_t + \gamma_i i_t + \gamma_z z_t + \varepsilon_t^g \tag{16}$$

$$b_t = \beta^{-1}[b_{t_1} + \varepsilon_t^g - \tau_t] \tag{17}$$

$$\tau_t = \phi_b b_{t-1} + \phi_g \varepsilon_t^g \tag{18}$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)[r_\pi \pi_t + r_y y_t] + \varepsilon_t^r \tag{19}$$

All variable and parameter interpretations may be found from either the GLV or SW papers directly. Detailed model discussions or derivations are omitted here for simplicity.

2 Empirical Methodology and Results

The model listed above is estimated using data for several U.S. macro indicators: y_t, c_t, i_t, n_t, w_t, π_t, and r_t. Data for all metrics were collected in real terms (where applicable) at a quarterly frequency from Q1 2010 through Q4 2022.[1] Inflation is computed using the PCE price index - the Fed's preferred metric (Yellen, 2015). Since the federal funds rate is frequently at its zero lower bound during this period, the Wu and Xia (2016) shadow federal funds rate is used to accurately capture the stance of monetary policy. Finally, given the importance of expectations in macro analysis (see Milani, 2023), one year ahead inflation expectations are collected from the Michigan Survey of Consumers and included in the estimation. The observation equations are as follows:

$$OBS_t = \begin{bmatrix} dlY_t \\ dlC_t \\ dlI_t \\ dlW_t \\ lHOURS_t \\ dlP_t \\ FFR_t \\ MSC_t^{1yr} \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{n} \\ \bar{\pi} \\ \bar{r} \\ 4\bar{\pi} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ n_t \\ \pi_t \\ r_t \\ \mathbb{E}_t \sum_{i=1}^{4} \pi_{t+i} + 4\varepsilon_t^m \end{bmatrix} \tag{20}$$

where dl represents 100 times the log difference, $\bar{\gamma}$ is the quarterly trend growth rate common to Y_t, C_t, I_t and W_t, $\bar{n}$ is set to zero since hours data is demeaned, $\bar{\pi}$ is the steady-state quarterly inflation rate, $\bar{r}$ is the steady-state quarterly interest rate, and ε_t^m is an exogenous measurement error. Calibrated parameters

[1] Unless specified otherwise, all data is collected from the FRED database.

Parameter	Value	Details
β	0.99	Discount rate
μ_p^{ss}	1.20	Steady state price mark-up
δ	0.025	Depreciation rate
γ_c	0.57	Consumption share
γ_i	0.25	Investment share
γ_z	0.10	Capital utilization share

Table 1: Calibrated Parameters

are presented in Table 1. The rest of the model parameters (structural and shocks) are estimated by a Bayesian MCMC Metropolis-Hastings algorithm using one chain of 500,000 draws with a 40% burn-in.[2] All parameters are identified and trace plots confirm parameter convergence. Priors for the estimated parameters (see Table 2) are chosen as per Smets and Wouters (2007) with the trends adjusted to accurately reflect the post-financial crisis period and shock deviations expanded. The posterior means for all parameters and shocks, as well as the marginal likelihood, is also presented in Table 2.

[2]See An and Schorfheide (2007), Fernández-Villaverde (2010), and Herbst and Schorfheide (2015) for an overview of Bayesian MCMC estimation methods pertaining to DSGE models.

Parameter	Description	Prior	Posterior
Structural Parameters			
h	Habit formation	$\mathbf{B}(0.70, 0.10)$	0.6180
σ_l	Inverse Frisch elasticity	$\mathbf{N}(2.00, 0.75)$	1.0709
ξ_p	Calvo factor - prices	$\mathbf{B}(0.50, 0.10)$	0.7612
ξ_w	Calvo factor - wages	$\mathbf{B}(0.50, 0.10)$	0.8339
ι_p	Price indexation	$\mathbf{B}(0.50, 0.15)$	0.1190
ι_w	Wage indexation	$\mathbf{B}(0.50, 0.15)$	0.5676
φ	SS capital adjustment elasticity	$\mathbf{N}(4.00, 1.50)$	1.5204
ψ	Capital utilization elasticity	$\mathbf{B}(0.50, 0.15)$	0.8047
α	Capital share of output	$\mathbf{N}(0.30, 0.05)$	0.3170
ϕ_b	Fiscal policy - debt	$\mathbf{\Gamma}(0.33, 0.10)$	0.3232
ϕ_g	Fiscal policy - spending	$\mathbf{\Gamma}(0.10, 0.05)$	0.0919
r_π	Taylor rule - inflation	$\mathbf{N}(1.50, 0.50)$	1.9289
r_y	Taylor rule - output	$\mathbf{N}(0.12, 0.05)$	0.1526
$\bar{\gamma}$	Economy trend	$\mathbf{N}(0.40, 0.10)$	0.4795
$\bar{\pi}$	SS inflation	$\mathbf{N}(0.60, 0.10)$	0.6987
$\bar{r}$	SS interest rate	$\mathbf{N}(0.75, 0.25)$	0.6820
Shock Processes			
Persistence			
ρ_b	Risk premium	$\mathbf{B}(0.50, 0.20)$	0.7482
ρ_w	Wage mark-up	$\mathbf{B}(0.50, 0.20)$	0.1576
ρ_i	Investment-specific tech.	$\mathbf{B}(0.50, 0.20)$	0.8772
ρ_p	Price mark-up	$\mathbf{B}(0.50, 0.20)$	0.9290
ρ_a	Total factor productivity	$\mathbf{B}(0.50, 0.20)$	0.4420
ρ_g	Fiscal spending	$\mathbf{B}(0.50, 0.20)$	0.9542
ρ_r	Monetary policy	$\mathbf{B}(0.50, 0.20)$	0.9145
Deviation			
σ_b	Risk premium	$\mathbf{\Gamma}^{-1}(0.30, 2.00)$	1.4887
σ_w	Wage mark-up	$\mathbf{\Gamma}^{-1}(0.30, 2.00)$	0.9462
σ_i	Investment-specific tech.	$\mathbf{\Gamma}^{-1}(0.30, 2.00)$	0.9202
σ_p	Price mark-up	$\mathbf{\Gamma}^{-1}(0.30, 2.00)$	0.1351
σ_a	Total factor productivity	$\mathbf{\Gamma}^{-1}(0.30, 2.00)$	0.5551
σ_g	Fiscal spending	$\mathbf{\Gamma}^{-1}(0.30, 2.00)$	0.5397
σ_r	Monetary policy	$\mathbf{\Gamma}^{-1}(0.30, 2.00)$	0.1494
σ_m	Measurement	$\mathbf{\Gamma}^{-1}(0.50, 2.00)$	0.2395
Marginal Likelihood (Modified Harmonic Mean)			-591.92

Table 2: Estimated Posterior Means and Marginal Likelihood

Note: For the priors, symbols represent distributions in the following manner: **B** - Beta, $\mathbf{\Gamma}$ - Gamma, **N** - Normal, and $\mathbf{\Gamma}^{-1}$ - Inverse Gamma. All prior distributions are presented with means and standard deviations in parentheses. 90% credible intervals are available upon request. Marginal likelihoods are computed using Geweke (1999)'s modified harmonic mean approach.

References

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