Investor Heuristics and Biases: Quantitative Effects on the U.S. Business Cycle

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This paper presents a medium-scale quantitative New-Keynesian DSGE model with financial intermediaries that incorporates a variety of behavioral modifications to investors. Investors in this model are subject to the anchoring and adjustment heuristic, endogenous confidence bias, and exogenous confidence bias. A Bayesian MCMC approach is utilized to estimate various iterations of this model, including shock process parameters, for the U.S. economy. The estimation aims to fit the model to six macroeconomic time series from 1988 through 2019, along with a measure of investors' expectations of future stock market performance. The estimated posterior means show that 18% or 31% of investor expectations arise from behavioral factors when modeled as exhibiting anchoring or confidence respectively. When exhibiting anchoring, investors are significantly pegged to 1-qtr and 1-year prior stock returns. In the model with endogenously propagated confidence, investors exhibit a confidence function that increases at a rate roughly halfway between a square and cubic root function. Both behavioral features are able to better fit the data as compared to the base model which includes only exogenous confidence shocks. In response to confidence shocks, the economy exhibits a dual-response fluctuation characterized by a large initial boom followed by a sustained recession. Models with behavioral features over-react to economic shocks compared to baseline, leading to more volatile business cycles.

Economists have long stressed the need for incorporating psychological factors as determinants of macro fluctuations. While such arguments date back at least as far as the 1920s, behavior-based analyses are only beginning to find a place in mainstream models of the U.S. economy. In the 1927 book *Industrial Fluctuations*, Pigou hypothesized that business cycles were largely driven by entrepreneurs experiencing waves of optimism or pessimism. In one of the field's seminal works, the *General Theory* written by Keynes in 1936, macro fluctuations are similarly attributed to entrepreneurs' "animal spirits" when faced with investment decisions. This paper models investors to exhibit heuristics and biases and aims to analyze and quantify the effects of such behavioral features on the

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U.S. business cycle.

The American Psychological Association defines a *bias* as "any deviation of a measured or calculated quantity from its actual (true) value, such that the measurement or calculation is unrepresentative of the item of interest" and *heuristics* as "rules-of-thumb that can be applied to guide decision-making based on a more limited subset of the available information." This paper incorporates such elements by proposing a departure from the standard rational expectations framework that is the common approach among most benchmark DSGE models today. Inspired by the arguments dating back to Pigou and Keynes, the mechanism by which investors in this economy form expectations will account for some common behavioral elements, thereby preventing evaluations of the future from being merely rooted in pure probability computations.

Literature in the field of social psychology clearly delineates that decision makers often exhibit behavior than cannot simply be explained by the classical rational actor model. While the entire set of possible departures from rational expectations is large, this paper focuses on a subset of particular behavioral features: anchoring and adjustment and confidence (both endogenous and exogenous).

Decision makers often exhibit the *anchoring and adjustment* heuristic. This is a phenomenon whereby an agent makes an estimate by starting at an initial value (the "reference point" or "anchor") which is then adjusted to arrive at the final estimate. Adjustments are typically insufficient and biased towards the anchor. In two seminal papers, Kahneman and Tversky (1974, 1979) explore a variety of psychological factors than impact agents making evaluations under uncertainty. Among them is the anchoring and adjustment heuristic. Numerous experiments from their papers corroborate the prevalence of this heuristic among agents and lead them to conclude that the "location of the reference point. . . emerge as critical factors in the analysis of decisions." In the specific context of investors, the anchor is usually a prior outcome (or a series of prior outcomes); returns generated in previous time periods are likely to affect evaluations of the future. Thaler and Johnson (1990) evaluate how risk-taking is affected by previous gains and losses through a series of experiments. They conclude that "real decision makers are influenced by prior outcomes" and finds evidence that such agents demonstrate "increased risk seeking in the presence of a prior gain." This paper will test the macroeconomic effects of the anchoring and adjustment heuristic.

Additionally, decision makers seem to bias expectations with their confidence rather than simply relying on a mathematical assessment of the distribution of potential outcomes. A common manifestation of confidence is the "better-than-average" effect: when asked to rate their relative skills, people seem to overestimate their ability relative to the average of the group (see Larwood and Whittaker (1997), Svenson (1981), and Alicke (1985)). Psychological underpinnings for confidence are typically attributed to three key factors: an illusion of control over outcomes, large commitments to positive outcomes, and establishing abstract reference points which renders performance comparisons difficult (Weinstein (1980); Alicke (1995)). While these psychological studies assess the effects of confidence pertaining to things such as motor skills or mortality, this phenomenon is also prevalent with respect to economic decision making (Camerer and Lovallo (1999)).

Theoretical and empirical research in behavioral finance has shown that confidence affects how investors and managers make financial decisions. For instance, Malmendier and Tate (2005, 2008, 2015) show that managers and CEOs overestimate the expected returns from investment projects; they overinvest when internal funding is abundant but reduce investment when relying on external funding. Note that this form of confidence differs from that hypothesized by Pigou or Keynes; here confidence may be built by an internal evaluation mechanism that is dependent on generated funds while the Pigouvian or Keynesian theory of optimistic or pessimistic waves and animal spirits suggests that confidence may also have an exogenous component that is unrelated to any economic fundamental. This paper will consider both of these forms of confidence: endogenous and exogenous. Prior literature has already shown that confidence can have business cycle effects and may have played a part during the financial crisis of the late 2000s.[1] In this paper, confidence is explicitly modeled in a proper DSGE framework to study its effects over modern U.S. economic history.

It is clear from prior literature, both in psychology and economics, that behavior plays a role in explaining economic outcomes. In the recent past, there have been several papers that incorporate such features in macro models. However, such attempts have been sparse with respect to financial agent behavior and unpacking the effects that imperfect rationality on the part of the financial sector can have on the macroeconomy. Caputo, et. al. (2010), using a calibration and simulation technique, find that macro fluctuations can be amplified if the financial accelerator mechanism is combined with a learning process. Similarly, Rychalovska (2016) also combines the financial accelerator model with adaptive learning but estimates the model using U.S. data. The paper also finds an amplified response of real variables to financial shocks but such responses are particularly tied to agents' perceptions of asset price persistence. Kedia (2022) combines a benchmark medium-scale financial frictions model with exogenous shocks to investor confidence and estimates the model using U.S. macro data. The paper finds that such shocks can trigger a business cycle: initially a shock that makes investors overconfident can trigger an expansion but such a boom is short-lived. Following a period of heavy over-investing, the economy goes into a prolonged recession. However, this paper does not account for an endogenously determined basis for confidence nor do any prior approaches study the effects of other behavioral features such as anchoring.

Section I presents the theoretical model utilized in this paper. Sections I.A to I.D describe the base model which is similar in most respects to the Gertler and Karadi (2011) medium-scale, monetary, dynamic, stochastic, general equilibrium model with a financial sector as well as a financial moral hazard friction. This model builds on and combines several prior approaches such as the benchmark models of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) that are used frequently in analytical papers as well as earlier financial frictions literature such as the Bernanke, Gertler, and Gilchrist (1999) financial accelerator model. Section I.E presents the mechanism by which investors form subjective expectations, as well as how such expectations account

[1] See: Kedia, 2022; Ho, et. al., 2016; Jlassi, et. al., 2014; and Abbes, 2013.

for behavioral biases and heuristics. This section also explains the mechanism by which biases and heuristics can affect the macroeconomy; deviations from rational expectations distort investors' perceptions of future returns, causing them to to increase or decrease leverage away from the optimal amount.

Section II describes the methodology employed by the paper to estimate the model with behavioral features. To gauge the real impact of these biases and heuristics, it is important to use actual U.S. data and see how well different behavioral features explain modern U.S. macroeconomics. This paper utilizes a Bayesian MCMC approach to fit the model to 5 standard U.S. macro series: real GDP, real personal consumption expenditures, real fixed private investment, inflation (GDP deflator), and the Federal Funds Rate as well as a measure of financial sector net worth and a survey measure of investors' expectations. Real U.S. domestic financial sector net worth is included so that the model may be tested on actual financial metrics, a feature that has been in lacking in prior approaches. Additionally, the use of actual expectations data in behavioral models, often in the form of surveys, has been recently emphasized by Coibion, Gorodnichenko, and Kamdar (2018) and Milani (2022). This paper utilizes actual survey data from the American Association of Institutional Investors as an observable in the estimation process to serve as a proxy for investor expectations. This allows a researcher to test modeled expectations against real data to see which behavioral factors, if any, play a role in explaining investors' thinking and to compare the relative importance of such factors. Several papers have successfully included survey expectations as observables in Bayesian macro models in the past but such approaches usually utilize data from the Survey of Professional Forecasters and aim to fit expectations of fundamental macrovariables. See Milani (2022) for a thorough literature review of such techniques. With respect to the use of expectations for model comparison, refer to Del Negro and Eusepi (2011) and Schorfheide (2005) who utilize inflation expectations to evaluate the importance of information regarding the inflation target. With respect to behavioral features, Milani (2017) uses expectations data to test whether persistence is driven by the endogenous features of the model or by agents' beliefs. The results highlight that inclusion of survey data favors a learning algorithm rather than structural sources of lags in fitting the data. Similarly, in a model where expectations depend on misspecified forecasting rules and myopia, Hajdini (2020) finds that behavioral features provide a better fit of the expectations data than persistence introduced through real rigidities. However, no attempt has been made to incorporate investor survey expectations in the evaluation of financial frictions models. This paper attempts to fill that gap in addition to contributing to the literature that uses survey expectations in evaluating behavioral macro models.

The results of the analysis are presented in section III. Both behavioral models significantly outperform the base model with exogenous confidence shocks in fitting the data with large increases in marginal likelihood. In the presence of subjectively determined investor expectations, traditional sources of persistence are found to be less important, mirroring results from prior behavioral macro studies such as Milani (2017). As per the MCMC estimation, investors anchor roughly 18% of their expectations to prior period ($T = 4$) returns with the one-quarter and one-year lagged returns being the most important anchors. If modeled as endogenously confident, investors form 31% of their expectations subjectively, with a confidence function that grows with net worth at a rate halfway between square and cubic root. Similar to Kedia (2022), all models exhibit dual-effect impulse responses to exogenous confidence shocks: due to a sudden wave of overconfidence, the economy experiences an immediate expansion driven by over-investing. However, this is followed by a large, sustained recession. These IRFs mirror the U.S. economy from the mid-2000s boom through the Great Recession. Behavioral features significantly amplify business cycles caused by both capital quality shocks as well as confidence shocks, as investors are cognitively affected by prevailing economic conditions and drift further away from rational expectations.

I. Theoretical Model

This section presents a model that incorporates a financial sector with a moral hazard friction into a state-of-the-art DSGE model with nominal rigidities. The model is similar to the Gertler and Karadi (2011) (henceforth 'GK2011') and includes several features from the benchmark DSGE models of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) (henceforth 'SW2007'), such as variable capital utilization, investment adjustment costs, habit formation, etc. These papers themselves build on the textbook treatment of business cycles that include a role for money and monetary policy such as Woodford (2003) and Galí (2008). To obtain an overview of the model, please refer to Appendix A where all the equilibrium conditions are summarized for convenience. This section begins by describing the economy under rational expectations with households detailed in section I.A, the financial sector in I.B, firms in I.C, and market-clearing and policy relations in I.D. The paper then details how investors' subjective expectations are modeled to account for behavioral heuristics and biases in section I.E.

A. Households

There is a $[0, 1]$ continuum of households that derive utility from consumption C_t that surpasses their stock of consumption habits from the past and suffer disutility by providing labor L_t. Their preferences are given by:

$$\max \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i} - hC_{t-1+i})^{1-\sigma}}{1-\sigma} - \chi \frac{L_{t+i}^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

where $0 < \beta < 1$ is the household's discount factor, $0 < h < 1$ is the degree of (external) habit formation, and $\sigma > 0, \varphi > 0$ are the inverses of the elasticities of intertemporal substitution and (Frisch) labor supply respectively. The parameter $\chi > 0$ is imposed so that the household devotes $1/3$ of its time to work at steady state.

Households save by lending funds to competitive financial intermediaries in the form of one-period riskless real bonds B_{t+1} which provide a return R_t from $t-1$ to t. The household receives a real wage W_t for each unit of labor supplied as well as earns profits

Π_t from its ownership of financial and non-financial firms (discussed in further detail in following sections). In addition, let T_t denote any net transfers made to the household. Then the household budget constraint may be written as:

$$C_t = W_t L_t + R_t B_t - B_{t+1} + \Pi_t + T_t \tag{2}$$

Households comprise two types of members: a fraction f of workers and $(1 - f)$ of bankers. Workers supply labor and return wages to the household as described above. Each banker manages a financial institution and also returns all earnings back to the household, thereby making households indirect owners of all banks. However, household deposits are made in banks that are not self-owned. It is assumed that there is perfect consumption insurance within the family. A family member may switch occupations, which is determined stochastically: a banker in any period remains so in the following period with independent probability θ. Therefore, the average survival time of a banker is given by $1/(1-\theta)$. On average, in every period $(1-\theta)f$ bankers become workers but since the reverse occurs with a similar probability, the relative proportion of bankers and workers stays the same. More details on the dynamics of new and existing bankers may be found in the next section.

Returning to the household's optimization problem, the household chooses C_t, L_t, and B_{t+1} so as to maximize its lifetime expected utility. Let ϱ_t denote the marginal utility of consumption; the intratemporal trade-off between consumption and labor may be computed as follows:

$$\varrho_t W_t = \chi L_t^{\varphi} \tag{3}$$

where

$$\varrho_t = (C_t - hC_{t-1})^{-\sigma} - \beta h \mathbb{E}_t (C_{t+1} - hC_t)^{-\sigma} \tag{4}$$

The intertemporal Euler equation is given by:

$$\mathbb{E}_t \beta \Lambda_{t,t+1} R_{t+1} = 1 \tag{5}$$

where

$$\Lambda_{t,t+1} = \mathbb{E}_t \frac{\varrho_{t+1}}{\varrho_t} \tag{6}$$

B. Financial Intermediaries

Banks[2] borrow money from households in the form of deposits which is in turn lent to non-financial firms. A banker j in time period t holds $S_{j,t}$ shares of goods producing firms that are each priced at Q_t and funds these investments by collecting $B_{j,t+1}$ deposits from households and via their own equity capital $N_{j,t}$. The banker's balance sheet is given by:

$$\underbrace{Q_t S_{j,t}}_{\text{Assets}} = \overbrace{B_{j,t+1}}^{\text{Liabilities}} + \underbrace{N_{j,t}}_{\text{Net Worth}} \tag{7}$$

Deposits from households, paid back at time $t+1$, earn a real gross return of R_{t+1}. The banker's assets earn the stochastic return R^k_{t+1} over this same period. Over multiple periods, the banker's net worth accrues from the difference between the earnings on assets and interest payments made to households on their borrowings:

$$N_{j,t+1} = R^k_{t+1} Q_t S_{j,t} - R_{t+1} B_{j,t+1} \tag{8}$$

Plugging-in the balance sheet relation from (7):

$$N_{j,t+1} = (R^k_{t+1} - R_{t+1}) Q_t S_{j,t} + R_{t+1} N_{j,t} \tag{9}$$

Let $m_{t,t+i}$ be the stochastic discount factor the banker utilizes at t to weight earnings at $t+i$. The banker must account for the probability of surviving into each future period as well as the intertemporal trade-off between current and future consumption. As such, the stochastic discount factor may be computed as:

$$m_{t,t+i} = \theta^i \beta^i \Lambda_{t,t+i} \tag{10}$$

The banker's objective at time t is to maximize terminal wealth:

$$V_{j,t} = \max\ \mathbb{E}_t \sum_{i=0}^{\infty} \beta(1-\theta) m_{t,t+1+i} N_{j,t+1+i} \tag{11}$$

which may be combined with the formulation for net worth from (9) as:

$$V_{j,t} = \max\ \mathbb{E}_t \sum_{i=0}^{\infty} \beta(1-\theta) m_{t,t+1+i} [(R^k_{t+1+i} - R_{t+1+i}) \\ Q_{t+i} S_{j,t+i} + R_{t+1+i} N_{j,t+i}] \tag{12}$$

where $V_{j,t}$ is the value of the bank at time t.

So long as the discounted, risk-adjusted, premium $(R^k_{t+1+i} - R_{t+1+i})$ the banker receives in any time period is positive, the banker will borrow infinitely from households to invest in firms. To prevent this, a moral hazard problem is introduced: in any time pe-

[2]The terms 'financial intermediaries' and 'banks' are used interchangeably in this paper. This is appropriate since there is only one type of financial institution in this model and it exhibits the most basic borrowing and lending mechanism that is common to virtually all investment and commercial banks.

riod, the banker can divert a fraction λ of its assets for personal benefit; however, if such a situation occurs, depositors will force a bankruptcy and recover the remaining $(1-\lambda)$ fraction of assets. As such, for depositors to provide the bank with funds, the following incentive compatibility constraint must hold:

$$V_{j,t} \geq \lambda Q_t S_{j,t} \tag{13}$$

Note that for easier computation, $V_{j,t}$ may be mathematically expressed as:

$$V_{j,t} = \nu_t Q_t S_{j,t} + \eta_t N_{j,t} \tag{14}$$

with

$$\nu_t = \mathbb{E}_t[(1-\theta)\beta \Lambda_{t,t+1}(R^k_{t+1} - R_{t+1}) + \beta \Lambda_{t,t+1}\theta x_{t,t+1}\nu_{t+1}] \tag{15}$$

$$\eta_t = \mathbb{E}_t[(1-\theta) + \beta \Lambda_{t,t+1}\theta z_{t,t+1}\eta_{t+1}] \tag{16}$$

where

$$x_{t,t+i} = \frac{Q_{t+i}S_{j,t+i}}{Q_t S_{j,t}} \tag{17}$$

is the gross growth rate of assets between periods t and $t+i$ and

$$z_{t,t+i} = \frac{N_{j,t+i}}{N_{j,t}} \tag{18}$$

is the gross growth rate of net worth in the same period. The term ν_t may be interpreted as the discounted expected marginal benefit to the banker of increasing asset holdings, while holding net worth fixed. Inversely, η_t may be interpreted as the discounted expected marginal benefit of increasing net worth, while keep asset holdings fixed.

Now when the incentive constraint binds, it may be expressed as:

$$\nu_t Q_t S_{j,t} + \eta_t N_{j,t} = \lambda Q_t S_{j,t} \tag{19}$$

which may be re-written to directly express the relation between assets and equity:

$$Q_t S_{j,t} = \frac{\eta_t}{\lambda - \nu_t} N_{j,t} = \phi_t N_{j,t} \tag{20}$$

It is clear from this expression that the nominal amount of shares the banker can hold is limited and is proportional to the bank's equity. The variable ϕ_t represents the ratio of the bank's assets to its equity and is referred to as the bank's *leverage ratio*. The evolution of the banker's net worth from equation (9) can now be adjusted to account for the leverage ratio:

$$N_{j,t+1} = [(R^k_{t+i} - R_{t+i})\phi_t + R_{t+1}]N_{j,t} \tag{21}$$

Additionally, as equation (20) shows, ϕ_t does not depend on any bank-specific factors. As such, we can aggregate across the individual banks to obtain the relation between the financial sector's demand for investments and its aggregate net worth:

$$Q_t S_t = \phi_t N_t \tag{22}$$

An aggregate financial sector net worth law of motion can now be computed. First, recall that a banker in any time period $t-1$ stays a banker in time period t with probability θ. This is done so as to prevent bankers from reaching a point where they are able to fund all investments simply by using their net worth. All exiting bankers will be replaced by workers that have transitioned into new bankers. Consequently, it is important to distinguish between the net worth of existing banks ($N_{e,t}$) and new banks ($N_{n,t}$) and recognize that the aggregate net worth must be computed as:

$$N_t = N_{e,t} + N_{n,t} \tag{23}$$

Since only a fraction θ of bankers survive from period $t-1$ to t, the law of motion for $N_{e,t}$ is calculated as:

$$N_{e,t} = \theta[(R^k_t - R_t)\phi_{t-1} + R_t]N_{t-1}\varepsilon^{N_e}_t \tag{24}$$

where $\varepsilon^{N_e}_t$ is an exogenous i.i.d. shock to bankers' existing net worth.

Bankers entering in any time period receive start-up funds; these funds take the form of a transfer of a small fraction of the assets accrued by the exiting bankers. Since bankers exit with the probability $(1-\theta)$, the total amount of assets held by exiting bankers at time t is $(1-\theta)Q_t S_{t-1}$. Assuming that the fraction of these assets that are transferred to new bankers is $\omega/(1-\theta)$, the aggregate net worth of new bankers is:

$$N_{n,t} = \omega Q_t S_{t-1} \tag{25}$$

Combine equations (23), (24), and (25) to compute the law of motion for aggregate financial sector net worth:

$$N_t = \omega Q_t S_{t-1} + \theta[(R^k_t - R_t)\phi_{t-1} + R_t]N_{t-1}\varepsilon^{N_e}_t \tag{26}$$

C. Firms

This section describes the production side of the economy as well as the investment dynamics that determine the stock price of capital. There are three types of firms: intermediate goods producers, capital repairers, and final retailers, each of which are described in further detail below.

INTERMEDIATE GOODS FIRMS. — These are competitive firms that produce goods that are eventually sold to retailers. At the end of every period, these firms acquire capital K_{t+1} to be used in production the following period. The intermediate firm does not face any capital adjustment costs and can simply sell its capital on the free market after it is used in the production process. The acquired capital is financed by obtaining the required funds from financial intermediaries. To do so, the firm issues shares S_t that act as claims on the K_{t+1} units of acquired capital; each share is valued at Q_t: the price of a unit of capital. So, $Q_t K_{t+1}$ is the total value of acquired capital and $Q_t S_t$ the total value of all claims against this capital. Under no arbitrage, $K_{t+1} = S_t$. Unlike financial intermediaries, the firms do not face any frictions in obtaining funds. These banks have perfect oversight over goods producers and can costlessly enforce any payoffs. All intermediate firms are identical so there is no need to index by producer type.

During each time period t, the intermediate firm produces output $Y_{m,t}$ by using capital K_t acquired in the prior period and the labor L_t supplied by workers from households. The firm can vary the quantity of capital it uses by adjusting the variable capital utilization rate U_t. The firm is subject to two exogenous AR(1) disturbances: A_t which denotes total factor productivity and ξ_t which is the quality of capital. Consequently, $\xi_t K_t$ is the effective quantity of capital that is available to the firm; this is similar in approach to Merton (1973) as ξ_t introduces a simple exogenous source of variation to the return on capital. Accounting for these factors and assuming constant returns to scale, the firm's production is given by:

$$Y_{m,t} = A_t (U_t \xi_t K_t)^{\alpha} L_t^{1-\alpha} \tag{27}$$

where $0 < \alpha < 1$ is the effective capital share of output. The market price of intermediate outputs is denoted by $P_{m,t}$. The firm pays a wage W_t per unit of labor and it is assumed that the replacement cost of used capital is one. Let $\delta(U_t)$ be a function that provides the rate of capital depreciation. The firm chooses the utilization rate and labor demand so as to maximize the sum of discounted expected future earnings:

$$\max \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} [P_{m,t+i} Y_{m,t+i} - W_{t+i} L_{t+i} - \delta(U_{t+i}) \xi_{t+i} K_{t+i}] \tag{28}$$

which leads to the following first-order conditions for the optimal choices of U_t and L_t:

$$P_{m,t}\alpha\frac{Y_{m,t}}{U_t} = \delta'(U_t)\xi_t K_t \tag{29}$$

$$P_{m,t}(1-\alpha)\frac{Y_{m,t}}{L_t} = W_t \tag{30}$$

Given that these intermediate firms are competitive, they earn zero profits in each period by paying out any ex-post return to capital back to the banks. This return must include the per unit value of leftover capital stock given by $(Q_{t+1}-\delta(U_{t+1}))\xi_{t+1}$. Consequently, $R_{k,t+1}$ is computed from:

$$Q_t R_{k,t+1} = P_{m,t+1}\alpha\frac{Y_{m,t+1}}{K_{t+1}} + (Q_{t+1} - \delta(U_{t+1}))\xi_{t+1} \tag{31}$$

Note that there is no explicit risk premium shock added to this model; exogenous variation in capital returns stems endogenously from the capital quality shocks discussed above. As such, the current price of capital will depend on expectations of the future path of these shocks. The specific functional form for the depreciation rate is given by:

$$\delta(U_t) = \delta_c + \frac{b}{1+\nu}U_t^{1+\nu} \tag{32}$$

where $\nu > 0$, b is the steady state value of the nominal marginal product of capital, and δ_c is set to maintain a steady state depreciation rate of 0.025.

CAPITAL PRODUCING FIRMS. — At the end of period t, these competitive firms buy capital from the intermediate firms which they refurbish at the aforementioned cost of one as well as construct new capital valued at Q_t. Both types of capital are then sold. There are flow adjustment costs that must be paid to produce new capital; there are no adjustment costs to renew capital. Households own capital producing firms and collect any accumulated profits. All capital firms are identical so there is no need to index by producer type. If $I_{n,t}$ is the amount of new capital produced and it is clear that $\delta(U_t)\xi_t K_t$ is the amount of refurbished capital, then the total amount of available capital in the economy is given by:

$$I_t = I_{n,t} + \delta(U_t)\xi_t K_t \tag{33}$$

Then the capital producing firm maximizes the discounted sum of future profits:

$$\max \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i}\left[(Q_{t+i}-1)I_{n,t+i} - f\left(\frac{I_{n,t+i}+I_{ss}}{I_{n,t-1+i}+I_{ss}}\right)(I_{n,t+i}+I_{ss})\right] \tag{34}$$

where $f(1) = f'(1) = 0$ and $f''(1) > 0$. The first-order condition provides the 'Q' relation for net investment:

$$Q_t = 1 + f(\cdot) + \frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}} f'(\cdot) - \mathbb{E}_t \beta \Lambda_{t,t+1} \left(\frac{I_{n,t+1} + I_{ss}}{I_{n,t} + I_{ss}} \right)^2 f'(\cdot) \tag{35}$$

The explicit functional form of $f(\cdot)$ is given by:

$$f(\cdot) = \frac{\eta_i}{2} \left(\frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}} - 1 \right)^2 \tag{36}$$

where $\eta_i > 0$ is the inverse elasticity of net investment to the price of capital.

New capital produced is added with existing capital to provide the following economy-wide capital accumulation equation:

$$K_{t+1} = \xi_t K_t + I_{n,t} \tag{37}$$

RETAIL FIRMS. — Monopolistically competitive retailers just re-package the goods produced by intermediate firms with retailer r using one unit of intermediate output $Y_{m,t}$ to produce a corresponding unit of retail output $Y_{r,t}$. The final economy output Y_t is a CES aggregate of the retail goods produced by a $[0, 1]$ continuum of differentiated retail firms:

$$Y_t = \left(\int_0^1 Y_{r,t}^{\frac{\epsilon-1}{\epsilon}} \, dr \right)^{\frac{\epsilon}{\epsilon-1}} \tag{38}$$

As per the cost minimization by final output users:

$$Y_{r,t} = \left(\frac{P_{r,t}}{P_t} \right)^{-\epsilon} Y_t \tag{39}$$

$$P_t = \left(\int_0^1 P_{r,t}^{1-\epsilon} dr \right)^{\frac{1}{1-\epsilon}} \tag{40}$$

Since the retailers' only input is the intermediate good, their marginal cost is the price of intermediate products $P_{m,t}$. Nominal rigidities in the form of Calvo (1983) pricing are now introduced: in each period t, a retail firm may freely adjust its price with probability $(1 - \gamma)$; the fraction γ of firms that cannot re-optimize simply index their prices to the lagged aggregate inflation rate. Retailers that can, must then choose the optimal price P_t^*

so as to maximize discounted future earnings:

$$\max \mathbb{E}_t \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{j=1}^{i} (1+\pi_{t+j-1})^{\gamma_p} - P_{m,t+i} \right] Y_{r,t+i} \tag{41}$$

where π_t is the rate of inflation in the economy and $0 < \gamma_p < 1$ is the degree to which prices are indexed to lagged inflation. The first-order conditions are given by:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{j=1}^{i} (1+\pi_{t+j-1})^{\gamma_p} - \frac{\epsilon}{\epsilon-1} P_{m,t+i} \right] Y_{r,t+i} = 0 \tag{42}$$

By applying the law of large numbers, the evolution of aggregate price level is:

$$P_t = [(1-\gamma)(P_t^*)^{1-\epsilon} + \gamma(\pi_{t-1}^{\gamma_p})^{1-\epsilon}]^{\frac{\epsilon}{1-\epsilon}} \tag{43}$$

D. Resource Constraints and Policy

Aggregate output is divided among consumption, investment (plus any adjustment costs), and government spending G_t. Consequently, the aggregate resource constraint is given by:

$$Y_t = C_t + I_t + G_t + \frac{\eta_i}{2} \left(\frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}} - 1 \right)^2 (I_{n,t} + I_{ss}) \tag{44}$$

with

$$G_t = G_{ss} g_t \tag{45}$$

where G_{ss} is steady state government spending and g_t is an exogenous AR(1) disturbance.

Monetary policy is assumed to be set by a monetary authority via a simple rule with interest rate smoothing that resembles a Taylor rule when linearized:

$$i_t = i_{t-1}^{\rho_i} \left[\frac{1}{\beta} \pi_t^{\kappa_\pi} \left(P_{m,t} \frac{\epsilon}{\epsilon-1} \right)^{\kappa_y} \right]^{1-\rho_i} \varepsilon_t^i \tag{46}$$

where $0 < \rho_i < 1$ is the interest rate smoothing parameter, $\kappa_\pi > 0$ is the inflation weight, $\kappa_y > 0$ is the output weight, and ε_t^i is an exogenous shock to monetary policy. Finally, to conclude the model, the Fisher equation relates the nominal interest rate, real interest rate, and inflation rate:

$$i_t = R_{t+1}\mathbb{E}\pi_{t+1} \tag{47}$$

E. Modeling Behavioral Features

This section first presents a generalized approach to incorporating behavioral effects into the financial sector of the model described above. Later, it will demonstrate how specific behavioral biases can be modeled within this framework. To begin, recall that banker j maximizes expected terminal wealth as shown in the following rational expectations equation:

$$V_{j,t} = \max\, \mathbb{E}_t \sum_{i=0}^{\infty} \beta(1-\theta)m_{t,t+1+i}[(R^k_{t+1+i} - R_{t+1+i}) \\ Q_{t+i}S_{j,t+i} + R_{t+1+i}N_{j,t+i}] \tag{48}$$

Now assume that the investor forms expectations about the future *subjectively*; such expectations are denoted by $\mathbb{E}^s_t$. Equation (48) may now be replaced by the banker's subjective maximization problem:

$$V_{j,t} = \max\, \mathbb{E}^s_t \sum_{i=0}^{\infty} \beta(1-\theta)m_{t,t+1+i}[(R^k_{t+1+i} - R_{t+1+i}) \\ Q_{t+i}S_{j,t+i} + R_{t+1+i}N_{j,t+i}] \tag{49}$$

Bankers are assumed to exhibit heuristics and biases primarily with respect to their evaluations of the future returns they can generate by investing in shares of goods producing firms: R^k_{t+1}. However, as noted in the introduction, a proper approach to incorporating behavioral features should also allow for the data to indicate that such elements play no role at all. As such, this model will assume that investors are not completely irrational; rather a fraction ζ of investors' assessments of future returns stems from behavioral factors. This parameter will be estimated later in the paper so as to allow the data to determine the degree to which investors exhibit behavioral biases. Subjective expectations of future returns may now be computed as follows:

$$\mathbb{E}^s_t R^k_{t+1} = \zeta R^s_{t+1} + (1-\zeta)\mathbb{E}_t R^k_{t+1} \tag{50}$$

where R^s_{t+1} is the assessment of future returns that is determined by any and all behavioral biases and heuristics. Note that until this point in the model, no specific behavioral feature has been incorporated within the mathematical framework. Any specific departure from rational agency or a combination of such features may now be incorporated within the computational framework for R^s_{t+1}. This paper will consider three such behavioral features: anchoring, endogenous confidence, and exogenous confidence. Note that financial firms in this model are all identical; for simplicity firm-specific variables are aggregated to an industry level. As a result, all behavioral features are assumed to affect the model at a financial *sector* level rather than for every individual bank. As such, a reader may prefer to interpret ζ as the percentage of investors in the economy that form expectations subjectively, similar to the heterogeneous expectations approach of Branch and McGough (2009).

ANCHORING. — As demonstrated by Tversky and Kahneman (1974), agents make estimates by starting from an initial reference point and then adjusting towards a final result. Often the adjustments are insufficient and the final estimate is biased towards the reference measure. Thaler and Johnson (1990) show that risk-taking agents are affected by prior outcomes. With this result in mind, anchoring is modeled as a dependence on prior returns generated by investors when investing in shares of goods producing firms. As such, under the presence of anchoring, for investors anchored to prior T period returns, R^s_{t+1} may be computed as follows:

$$R^s_{t+1} = \rho_1 R^k_t + \rho_2 R^k_{t-1} + \cdots + \rho_T R^k_{t+1-T} \tag{51}$$

Note that it would be unrealistic to assume that investors would still be exhibiting such biases at steady state where the values of all variables never deviate. At such a point, there would be no uncertainty of the future and therefore investors would be unlikely to exhibit biased evaluations of economic conditions. As such, the restriction $\sum_{j=1}^{T} \rho_j = 1$ is imposed to ensure that the equilibrium under subjective expectations converges to the rational expectations equilibrium at steady state.

ENDOGENOUS CONFIDENCE. — Agents may also be under/over-confident of their own ability to generate returns. Substantial evidence, both theoretical and empirical, demonstrates that economic agents suffer from this bias as documented in detail in the introduction to this paper. To model confidence, this paper relies on the key result from Malmendier and Tate (2005) that economic agents' confidence in their own investing prowess increases proportionally with their own internal funding. Banks in GK2011 have access to internal and external funding in the form of their own net worth and deposits from households respectively. As such, confidence is considered to be a function of net worth so that subjective returns may be computed as:

$$R^s_{t+1} = f(N_t) R_{t+1} \tag{52}$$

where $f(0) = 1$, $f' > 0$, and $f'' < 0$. Given this formulation, when the bank has no net worth at all, the investor has no confidence in generating returns and assumes that the best rate available is the risk-free rate R_{t+1}. Once the investor starts generating net worth, confidence increases, albeit at a decreasing rate. This confidence is then used to scale up from the risk-free rate. Consider the particular functional form for $f(\cdot)$ to be used in the estimation process:

$$f(N_t) = \chi_c \left(\frac{1}{\chi_c^{1+\zeta_c}} + N_t \right)^{\frac{1}{1+\zeta_c}} \tag{53}$$

where $\chi_c > 0$ and $\zeta_c > 0$ is the elasticity of the confidence function. The parameter χ_c is imposed so that $R^s_{t+1} = R^k_{t+1}$ at steady state for similar reasons as detailed above during the discussion on anchoring.

EXOGENOUS CONFIDENCE. — Investors may be subject to a sudden exogenous wave of optimism or pessimism. This is represented by a shock process that is appended directly on to the expectation formation mechanism:

$$\mathbb{E}^s_t R^k_{t+1} = \zeta R^s_{t+1} + (1-\zeta)\mathbb{E}_t R^k_{t+1} + \varepsilon^{\zeta}_t \tag{54}$$

where ε^{ζ}_t is an exogenous i.i.d. shock process that is distributed normally with a mean of 0 and standard deviation σ^{ζ}.

To understand the effects that such behavioral elements have on the model, recall from equation (14) that the bankers' value function is computed as:

$$V_t = \nu^s_t Q_t S_{j,t} + \eta_t N_t \tag{55}$$

Note that unlike the base model, ν^s_t is a function of the subjective returns evaluation: $\mathbb{E}^s_t R^k_{t+1}$. Also recall the calculation of banks' leverage ratios from equation (20):

$$\phi^s_t = \frac{\eta_t}{\lambda - \nu^s_t} \tag{56}$$

The degree to which banks are leveraged is now also dependent on $\mathbb{E}^s_t R^k_{t+1}$ via ν^s_t. If a banker, as a consequence of a behavioral bias or heuristic, has a subjective returns assessment $\mathbb{E}^s_t R^k_{t+1} > \mathbb{E}_t R^k_{t+1}$, then it follows that $\nu^s_t > \nu_t$. As is evident from equation (56), the leverage ratio implied by the behavioral model will be higher than the corresponding optimal leverage ratio from the base model ($\phi^s_t > \phi_t$). Consequently, the financial sector could be over/under-leveraged as a result of subjective returns assessments.

II. Data and Methodology

The various behavioral models presented in section I.E are estimated via Bayesian MCMC techniques[3] to fit data for six quarterly macroeconomic U.S. time series: log

[3]See An and Schorfheide (2007), Fernández-Villaverde (2010), and Herbst and Schorfheide (2015) for an overview of Bayesian MCMC estimation methods pertaining to DSGE models.

difference of real GDP, log difference of real personal consumption, log difference of real private investment, demeaned log difference of real financial sector net worth, inflation (log difference of GDP deflator), and the federal funds rate. The raw data series, prior to conversion to growth rates, are presented in Figure 1. The inclusion of financial data in the form of net worth is a key feature of this paper's estimation process and has been missing from many of the prior empirical papers that use a Bayesian approach to fit financial frictions models to macro data. Naturally, such an inclusion enriches the results of the paper by providing the model the ability to fit macrofinance data in addition to the traditional measures of the macroeconomy.

Additionally, as discussed in the introduction, prior empirical approaches in this area of study have largely ignored expectations data. Since the primary innovation of this paper is the varied modeling of investor expectations, it is important to include expectations data to see if the model with such features provides a better fit and thereby better evidence that investors indeed exhibit such behavior. To derive investor expectations, this paper utilizes data from the Investor Sentiment Survey conducted by the American Association of Individual Investors. In this survey, respondents are asked the following question each week: "What Direction Do AAII Members Feel The Stock Market Will Be In The Next 6 Months?" They may answer only by selecting either "Bullish", "Bearish", or "Neutral". The average quarterly spread between the respondents answering "Bullish" versus "Bearish" is used as a proxy for investor expectations over the coming two quarters. This data series may also be viewed in Figure 1. This approach closely follows the method used by Greenwood and Shleifer (2014) who similarly use a bull-bear spread as their benchmark measure for expectations but utilize the Gallup survey instead of the AAII. The AAII survey offers a few benefits over the Gallup survey: (1) it is easier to access as it does not require a paid subscription, (2) it began in 1987, offering more observations than the Gallup survey which began in 1996, and (3) it asks respondents to present their sentiments for the upcoming 6-months instead of a full year as in Gallup, allowing for a nearer term outlook.

The final dataset spans Q1 1988 through Q4 2019: starting from the first full year where AAII survey data is available and proceeding until the start of the COVID-19 pandemic; this period also roughly corresponds to the modern U.S. macroeconomy with active monetary policy. The measurement equation used in the estimation procedure for the standard non-expectations macro data is given by:

$$OBS_t = \begin{bmatrix} dlY_t \\ dlC_t \\ dlI_t \\ dlN_t \\ dlP_t \\ i_t \end{bmatrix} = \begin{bmatrix} \bar{\chi} \\ \bar{\chi} \\ \bar{\chi} \\ 0 \\ \bar{\pi} \\ \bar{i} \end{bmatrix} + \begin{bmatrix} \log Y_t/Y_{t-1} \\ \log C_t/C_{t-1} \\ \log I_t/I_{t-1} \\ \log N_t/N_{t-1} \\ \log P_t/P_{t-1} \\ i_t \end{bmatrix} \tag{57}$$

where dl represents 100 times the log difference, $\bar{\chi}$ is the quarterly trend growth rate common to Y_t, C_t, and I_t, $\bar{\pi}$ is the steady-state quarterly inflation rate, and $\bar{i}$ is the

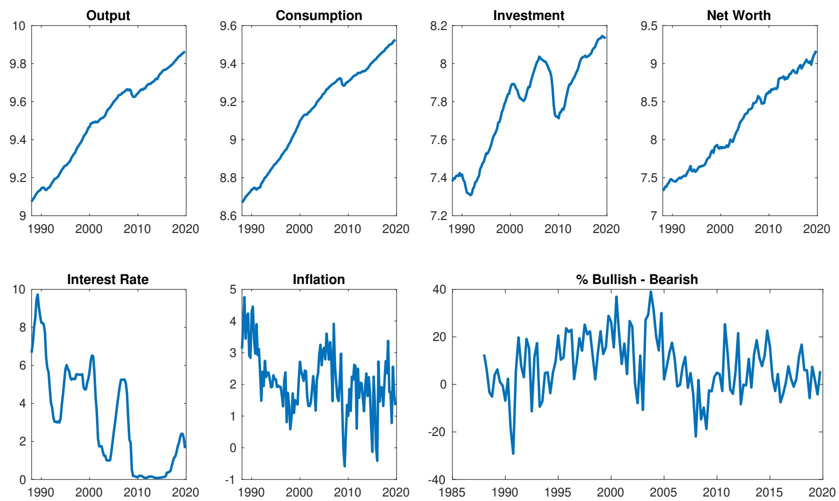

FIGURE 1. ORIGINAL DATA SERIES

Note: Output, consumption, investment, and net worth are converted to real terms and expressed in log-levels. Values of inflation and interest rate are annualized and expressed in levels. The bullish-bearish spread is computed for every observation from the AAII survey and the average quarterly value is reported.

steady-state quarterly interest rate.

An additional measurement equation is needed for the investor expectation data. Note that since the AAII survey data asks investors about their sentiments towards the stock market two quarters ahead, this measurement equation will require a connection between the bullish-bearish spread and the sum of the subjective returns expectations over the upcoming two quarters using a standard regression equation:

$$SPREAD_t = \beta_0 + \beta_1(R^s_{t+1} + R^s_{t+2}) + \varepsilon^{obs}_t \tag{58}$$

where ε^{obs}_t is a normally distributed measurement error.

Some structural parameters are calibrated to the same values utilized by GK2011. These parameters are presented in Table 1. The remaining parameters are estimated using a standard Bayesian MCMC procedure. The priors for these selected parameters are set based on standard choices in the empirical macro literature and may be found in Table 2. Habit formation (h), intertemporal elasticity of substitution (σ), Calvo pricing (γ), Taylor rule coefficients (κ_π and κ_y), output trend ($\bar{\gamma}$), and inflation trend ($\bar{\pi}$) all follow the same distributions as SW2007. In the cases of σ, γ, and κ_y, standard deviations are slightly elevated from their corresponding values in SW2007. Owing to varied estimates of price indexation (γ_p) across the literature, this parameter is assigned an uninformative uniform prior. Investment adjustment mechanics in this model also differ from

Parameter	Value	Details
β	0.99	Discount rate
φ	0.276	Inverse Frisch elasticity of labor supply
θ	0.972	Bankers' survival rate
α	0.33	Effective share of capital
υ	7.2	Elasticity of marginal depreciation wrt utilization rate
ε	4.167	Elasticity of goods substitution
$\bar{\delta}$	0.025	Steady state depreciation rate
$\bar{\phi}$	4	Steady state leverage ratio
$\bar{R^k} - \bar{R}$	0.0025	Steady state market premium
$\bar{L}$	1/3	Steady state labor supply
G/Y	0.2	Steady state government spending ratio

TABLE 1—CALIBRATED PARAMETERS

prior approaches such as SW2007 so the elasticity of investment adjustment is assigned a wider prior that follows a Gamma distribution with mean 4.00 and deviation 1.50. Unlike SW2007, interest rate trend ($\bar{i}$) is modeled separately from the economy and output trends so it is assigned its own prior which follows a Normal distribution with mean 0.75 and deviation 0.10. The regression coefficients (β_0 and β_1) that fit investor expectations to the survey data are both distributed Normal with means 0 and 1 and deviations 0.30 and 0.20 respectively. The prior for the degree of subjectivity (ζ) is kept uninformative with a Uniform distribution. Anchoring coefficients ($\rho_1, \ldots, \rho_3$) are assigned (relatively) Minnesota priors: all coefficients are distributed Normal with mean 0 and deviation 0.25 except the one-quarter lag which is assigned a mean of 0.9. Recall that these coefficients must all add up to unity, suggesting an implied prior mean of 0.1 on the 1-year lag. The elasticity of the confidence function (ζ_c) is given a wide prior distributed Gamma with mean 2 (corresponding to a cubic root relation between net worth and confidence) and deviation 1.50.

The Bayesian algorithm proceeds as follows. First, the mode of the posterior distribution is estimated by maximizing the log of the posterior function; the posterior is computed as the product of the prior information of non-calibrated parameters and the likelihood of the data described above. Secondly, a Metropolis-Hastings computational algorithm comprising two MCMC chains of 500,000 draws each (enough to achieve convergence) is utilized to map a complete posterior distribution for all estimated parameters. This process is used to estimate various iterations of the base model with various behavioral features added (more details in the next section). Note that all estimated parameters are identified from the data in all versions of the model. The estimated posterior means are used to compute IRFs to the various shocks within the model. The results from these analyses are presented in the following section.

Parameter	Description	Prior	Exo. Confidence K2023(a)	Anchoring ($T = 4$) K2023(b)	End. Confidence K2023(c)
h	Habit formation	$\mathbf{B}(0.70, 0.10)$	0.8389	0.7915	0.8107
σ	IES	$\mathbf{\Gamma}(1.50, 1.00)$	0.5712	0.8369	0.7706
γ	Calvo factor	$\mathbf{B}(0.50, 0.15)$	0.8172	0.9001	0.9009
γ_p	Price Indexation	$\mathbf{U}(0.00, 1.00)$	0.7720	0.0525	0.0578
η_i	Inv. Adjustment	$\mathbf{\Gamma}(4.00, 1.50)$	4.2032	0.1793	0.1765
κ_π	Taylor Rule	$\mathbf{N}(1.50, 0.25)$	2.1320	1.8516	1.7866
κ_y	Taylor Rule	$\mathbf{N}(0.13, 0.06)$	0.3536	0.0547	0.0584
$\bar{\chi}$	Trend	$\mathbf{N}(0.40, 0.10)$	0.7426	0.9305	0.9360
$\bar{\pi}$	Trend	$\mathbf{N}(0.60, 0.10)$	0.5654	0.7707	0.7560
$\bar{i}$	Trend	$\mathbf{N}(0.75, 0.10)$	0.7043	0.7907	0.6725
β_0	Observation	$\mathbf{N}(0.00, 0.30)$	0.0761	0.0747	0.0766
β_1	Observation	$\mathbf{N}(1.00, 0.25)$	0.0167	0.0405	0.0393
ζ	Subjectivity	$\mathbf{U}(0.00, 1.00)$	-	0.1755	0.3144
ρ_1	Anchoring	$\mathbf{N}(0.90, 0.25)$	-	0.6835	-
ρ_2	Anchoring	$\mathbf{N}(0.00, 0.25)$	-	-0.2099	-
ρ_3	Anchoring	$\mathbf{N}(0.00, 0.25)$	-	0.1219	-
ρ_4	Anchoring	-	-	0.4045	-
ζ_c	Confidence	$\mathbf{\Gamma}(2.00, 1.50)$	-	-	1.5549

TABLE 2—POSTERIOR MEANS OF STRUCTURAL PARAMETERS

Note: The table reports posterior means for the following models: under rational expectations [K2023(a)], with $T = 4$ quarters of anchoring [K2023(b)], and with endogenous confidence [K2023(c)]. All models include exogenous confidence shocks to investors' expectations formation process. For the priors, symbols represent distributions in the following manner: $\mathbf{B}$ - Beta, $\mathbf{\Gamma}$ - Gamma, $\mathbf{U}$ - Uniform, and $\mathbf{N}$ - Normal. All prior distributions are presented with means and standard deviations in parentheses except $\mathbf{U}$ which shows lower and upper bounds. Posterior means have been computed over two chains of 500,000 Metropolis-Hastings draws each and after a 40% burn-in. The data sample spans from Q1 1988 to Q4 2019.

III. Results

A. *Posterior Estimates*

Table 2 shows the mean of the posterior distribution of structural parameters obtained from the Bayesian estimation procedure described above. The table compares the results from estimating the following variations of the model: exogenous confidence ("K2023(a)"), 4 quarters of anchoring ("K2023(b)"), and endogenous confidence ("K2023(c)"). Note that no comparison is made to the base GK2011 model; this exercise is conducted in a prior paper: Kedia (2022), which already shows that a model including exogenous confidence shocks offers several benefits over the base financial frictions model. Another advantage to this method is that it allows for all versions of the K2023 model to be tested on the same dataset: 7 time series with a corresponding 7 shock processes. The table only shows the mean and not the dispersion of the estimated posteriors; the 10% and 90% credible intervals of each parameter may be found by viewing Table B1 in the appendix.

The key finding is that the inclusion of subjective expectations is important; when considering anchored investors, subjectivity (ζ) forms approximately 18% of their ex-

Parameter	Description	Prior	Exo. Confidence K2023(a)	Anchoring ($T = 4$) K2023(b)	End. Confidence K2023(c)
Persistence					
ρ_a	Technology	$\mathbf{B}(0.50, 0.20)$	0.3929	0.5243	0.5161
ρ_g	Govt. Spending	$\mathbf{B}(0.50, 0.20)$	0.9489	0.9642	0.9651
ρ_i	Monetary Policy	$\mathbf{B}(0.50, 0.20)$	0.3973	0.0932	0.1105
ρ_ξ	Capital Quality	$\mathbf{B}(0.50, 0.20)$	0.2643	0.0123	0.0124
ρ_s	Ex. Confidence	$\mathbf{B}(0.50, 0.20)$	0.3344	-	-
Deviation					
σ_a	Technology	$\mathbf{\Gamma}^{-1}(0.30, 1.00)$	0.0394	0.0513	0.0533
σ_g	Govt. Spending	$\mathbf{\Gamma}^{-1}(0.30, 1.00)$	0.0419	0.0443	0.0440
σ_i	Monetary Policy	$\mathbf{\Gamma}^{-1}(0.30, 1.00)$	0.0371	0.0371	0.0371
σ_ξ	Capital Quality	$\mathbf{\Gamma}^{-1}(0.30, 1.00)$	0.0377	0.0387	0.0390
σ_{Ne}	Net Worth	$\mathbf{\Gamma}^{-1}(0.30, 1.00)$	0.2172	0.0514	0.0516
σ_s	Ex. Confidence	$\mathbf{\Gamma}^{-1}(0.30, 1.00)$	2.0011	0.9318	0.9424
σ_{obs}	Measurement Error	$\mathbf{\Gamma}^{-1}(0.30, 1.00)$	0.1112	0.1199	0.1191

TABLE 3—POSTERIOR MEANS OF SHOCK PROCESSES

Note: The table reports posterior means for the following models: under rational expectations [K2023(a)], with $T = 4$ quarters of anchoring [K2023(b)], and with endogenous confidence [K2023(c)]. All models include exogenous confidence shocks to investors' expectations formation process. For the priors, symbols represent distributions in the following manner: $\mathbf{B}$ - Beta and $\mathbf{\Gamma}^{-1}$ - Inverse Gamma. All prior distributions are presented with means and standard deviations. Posterior means have been computed over two chains of 500,000 Metropolis-Hastings draws each and after a 40% burn-in. The data sample spans from Q1 1988 to Q4 2019.

	Exo. Confidence K2023(a)	Anchoring (T=4) K2023(b)	End. Confidence K2023(c)
Marginal Likelihood	-2531.47	-2426.31	-2422.43

TABLE 4—MODEL COMPARISON

Note: Marginal likelihoods are computed using the Geweke (1999) modified harmonic mean approach.

pectations of future returns. Within the expectation formation process, it is clear that anchoring plays an important role. The first (ρ_1) and fourth (ρ_4) quarters of anchoring have large estimated values of 0.68 and 0.40. The finding that the one year prior returns exhibit high persistence is in accordance with the empirical results from Figure 6 and Table 3 of Greenwood and Shleifer (2014), who also find that one year lagged stock returns are an important determinant of investor expectations. The degree of subjectivity is an even higher 31% when modeling investors as endogenously confident. The value of ζ_c, the parameter that governs the shape and rate of diminishment of the confidence function, is estimated to be 1.55; this corresponds to a confidence function that depends on net worth in a manner that is roughly halfway between square and cubic root. Interestingly, the posterior from both behavioral models look similar. Means across most parameters are roughly the same in both behavioral iterations of the model. This is particularly fascinating given the different approaches taken in modeling these two different biases, perhaps suggesting that both anchoring and overconfidence are capturing the same cognitive element within the data.

The analysis between these models offers several other insights that support the inclusion of anchoring or endogenous confidence. The need for several structural sources of persistence falls precipitously in the presence of behavioral elements. The degree of price indexation (γ_p) falls drastically from 0.77 in K2023(a) to 0.05 and 0.06 in K2023 (b) and (c) respectively, virtually rejecting the need for any indexation of prices to prior lags. Interestingly, the coefficient that governs the importance of investment adjustment costs falls drastically from 4.20 in K2023(a) to approximately 0.18 in both the behavioral models. This indicates that the sluggishness of investment responses is likely due to investors' own cognitive pegs to prior return lags or to their confidence via their net worth rather than mechanical hindrances to investment updating. This result corroborates findings from Milani (2017) that the inclusion of behavioral features lessens the need for structural sources of persistence, elasticity of investment adjustment costs in particular, that often have poor micro-founded evidence.

The degree of habit formation (h) remains relatively unchanged across all models: 0.84 in (a), 0.79 in (b), and 0.81 in (c). Given that habit formation is usually included even in rudimentary DSGE models of the macroeconomy, this is further confirmation of its importance to fitting macro data; habits continue to remain necessary even in the presence of behavioral features. The intertemporal elasticity of substitution (σ) moves closer to a log-utility specification in the behavioral models, increasing from 0.5712 in (a) to 0.84 in (b) and 0.77 in (c). The level of price stickiness (γ) is high in the base model with a value of 0.82; this increases further to 0.90 in both (b) and (c). This closely matches estimated price stickiness from Milani (2017), suggesting that Calvo pricing plays an important role in behavioral models. In both behavioral models, monetary policy is less responsive to both inflation and output. The inflation targeting coefficient (κ_π) falls from 2.13 to 1.85 and 1.79, values that are much closer to usual calibrations of this parameter. In the base model, the coefficient of output targeting (κ_y) is unreasonably high at 0.35 whereas this coefficient is significantly smaller in K2023(a) and (b) at 0.06.

The posterior distribution of shock process parameters is displayed in Table 3. Again, the table only shows the mean and not the dispersion of the estimated posteriors; the 10% and 90% credible intervals of each shock process persistence and deviation may be found by viewing Table B2 in the appendix. Similar to most empirical estimates, the persistence of government spending is high (above 0.94) and both the AR coefficient as well as the shock deviation are relatively similar across all models. Other shocks show marked differences between the base and behavioral models. Technology shocks are more persistent: 0.39 in K2023(a) compared to 0.52 in both K2023(b) and K2023(c). Monetary policy is significantly less pegged to past interest rates with its persistence falling from 0.40 in (a) to 0.09 and 0.11 in (b) and (c) respectively. The deviation of monetary shocks remain virtually unchanged across all models. Capital quality shocks are remarkably less persistent: 0.26 in (a) versus 0.01 in the behavioral models. The deviation of these shocks stays fairly similar. Interestingly, as discussed in more detail in the next section, the effect of this shock on the economy is often larger in the behavioral models as the

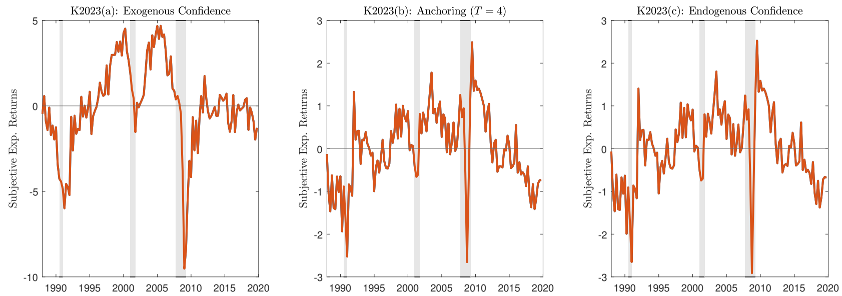

FIGURE 2. IMPLIED SUBJECTIVE EXPECTED RETURNS

Note: Implied expected returns are reported for the models with $T = 4$ quarters of anchoring [K2023(a)] and endogenous confidence [K2023(b)] from left to right respectively. Grey shaded bars represent official NBER-dated recessions.

one-time shock is propagated more vigorously through the model dynamics owing to its interaction with investors' subjective expectations. The persistence of exogenous confidence shocks, present only in the base model, is estimated to be 0.33. The volatility of such shocks are markedly different; K2023(a) requires large confidence shocks to fit the data. This is no longer the case when investor expectations are modeled explicitly. The deviation of these shocks falls from 2.00 in (a) to only 0.93 and 0.94 in (b) and (c) respectively. Net worth shocks are never modeled with AR coefficients; their deviations are also lower, falling from 0.22 in the base model to 0.05 in the behavioral extensions.

Behavioral features markedly improve the model's ability to fit the macro time series data. Table 4 shows the marginal likelihood for all three models, computed using the Geweke (1999) modified harmonic mean approach. K2023(a) has an estimated marginal likelihood of -2531 versus values of -2426 and -2422 for K2023(b) and K2023(c) respectively: a stark increase in likelihood of over 100 points. In the presence of anchoring and endogenous confidence, along with the inclusion of financial sector net worth and expectations as observables, the addition of behavioral features now offers clear improvements in model fit.

Does the model generate realistic subjective returns? Figure 2 shows the investors' model-implied smoothed subjective expected returns from all three models. As noted above, the parameter estimates from K2023(b) and (c) are similar which results in subjective returns that are equivalently similar. The graph shows that investors' subjective expectations mirror optimistic/pessimistic cycles in modern U.S. economic history across all models. For instance, the mid-2000s period prior to the Great Recession, characterized by highly leveraged financial institutions and the inflation of asset bubbles (especially in the housing sector), corresponds to investors' expectations being significantly elevated. In K2023(a), these expectations are at the highest level during the entire sample while in the behavioral models they are second only to the period immediately after the end of the financial crisis. Unsurprisingly, investors' expectations are

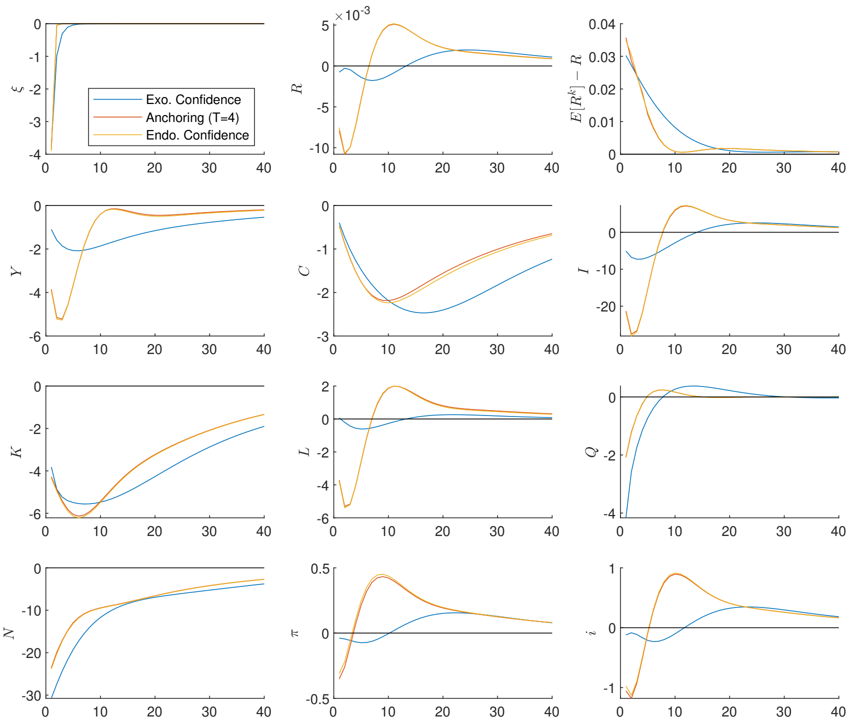

FIGURE 3. IMPULSE RESPONSES TO A NEGATIVE CAPITAL QUALITY SHOCK

Note: The figure displays mean impulse responses across Metropolis-Hastings draws. All impulse response values are expressed as percentage deviation from steady-state. Values for R, $\mathbb{E}[R^k] - R$, π, and i have been annualized.

also affected by the state of the business cycle; pessimism increases during recessions with lowest expected returns corresponding to the dot-com bubble burst and the financial crisis. The model with only exogenous confidence shocks generates the most volatile expectations, likely due to the lack of any disciplining element in the way expectations are modeled in K2023(a). Under anchoring and confidence, backward-looking behavior prevents investors from over-reacting to economic events. Note that this correlates with a famous result in macro expectations: Coibion and Gorodnichenko (2015) find that aggregate surveys of institutional respondents under-react to news; this is confirmed under the behavioral models presented here.

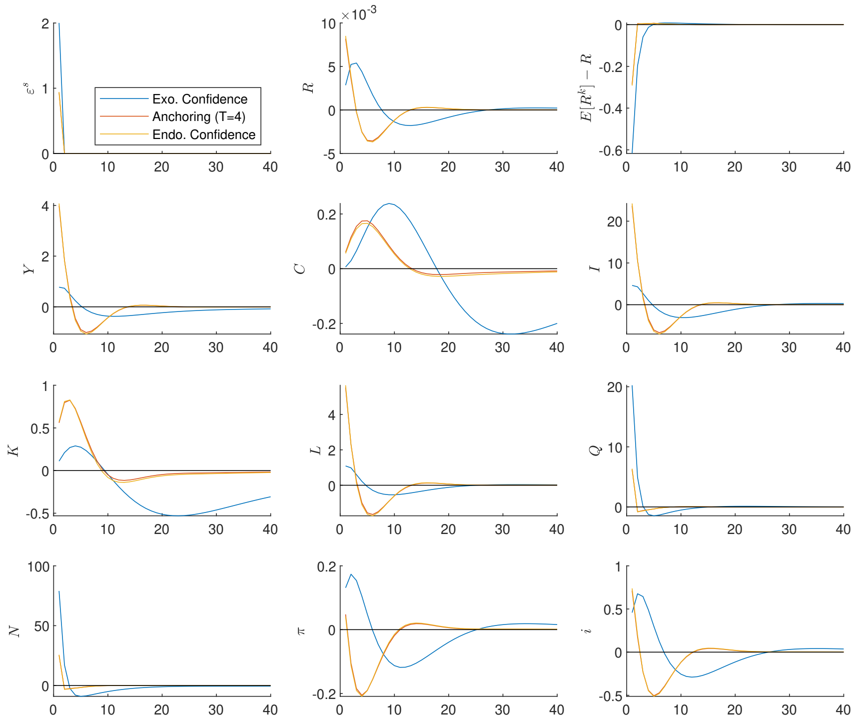

FIGURE 4. IMPULSE RESPONSES TO AN OVERCONFIDENCE SHOCK

Note: The figure displays mean impulse responses across Metropolis-Hastings draws. All impulse response values are expressed as percentage deviation from steady-state. Values for R, $\mathbb{E}[R^k] - R$, π, and i have been annualized.

B. Impulse Responses

This paper is primarily concerned with the effects of two shocks: capital quality and confidence. In this section, impulse responses to both these shocks are discussed, beginning with capital quality. Figure 3 shows the comparative impulse responses of key model variables to a one-period, 1 standard deviation, negative shock to capital quality. As expected, the economy enters a prolonged recession following the shock in both models. In a similar manner to the mechanism described in GK2011, when the negative capital quality shock occurs the effective capital in the economy falls. Since the financial sector is invested in this capital and holds the corresponding shares as assets on their balance sheets, banks experience a sudden and large fall in asset holdings. To maintain its balance sheet constraints under leverage, the bankers' net worth falls along with their demand for more assets. As the demand for investments in capital falls, the share price/price of capital also falls. Due to the decreased effective capital as well as investment in capital, firms cannot produce as much which causes a drop in output and thereby a recession. Consumption is also lowered as firms curtail their labor demand and banks reduce the interest they pay on deposits from households. The economic recovery is driven by investment, which rises above steady state roughly 3 years after the initial shock. However, even the increased investment level cannot compensate for the decline in consumption and as a result, output stays below steady state even at a horizon of 10 years following the shock.

It is interesting to note that the magnitude of the capital quality shock is *amplified* in the K2023(b) and (c) models, although the recession is not as prolonged. Since capital quality is directly linked with the return to capital and since investors draw expectations of the future using multiple prior quarter returns, the effect of the shock lingers for multiple periods under anchoring in K2023(b). In K2023(c), the massive and prolonged fall in net worth reduces investor confidence for multiple periods, causing a larger decline in investment than normal. As a result, investors over-react to the immediate shock and only later adjust their expectations in both models, the cognitive effects of a shock affecting the economy even in quarters when the economy is not undergoing a shock. The inclusion of behavioral features may also help explain the "missing deflation" puzzle following the financial crisis. Owing to the recession, economists expected an accompanying deflationary episode that never materialized; in the IRFs from K2023(b) and (c), deflation occurs very briefly at the start of the recession and in turn leads into an *inflationary* episode, mirroring the actualized recession of 2008-09.

Figure 4 shows the impulse responses of key variables to a one-period, 1 standard deviation, positive shock to investor confidence, i.e. 1 standard deviation "overconfidence" among financial intermediaries. In the period of impact, investor overconfidence is able to stimulate the economy above its steady state. As investors are suddenly overconfident in their ability to generate returns, their demand for shares in goods producing firms increases. This significantly raises the share price Q_t; note that this exogenous shock is able to create a stock price bubble: a sharp increase in stock prices without any actual change to macro fundamentals. As the investors pour more funds into capital, firms want to increase production. Their demand for labor increases which raises wages as well as the labor supply. This causes the economy to go into an expansion. Consumption is initially slow to follow the increases in output; since banks want more deposits to fund more investments, the interest rate on bank deposits increases and households choose to save their extra labor income rather than consume. This boost in output is short-lived as the positive effects of overconfidence dissipate roughly 1 year after the point of impact.

After the initial boost wanes, the economy enters a prolonged recession. Since banks' investment decisions are not based on economic fundamentals, they choose to increase investments at a period when the market premium is below steady state. As banks' net worth evolves proportional to the premium, it starts to fall and goes below steady state. Meanwhile, owing to lowered interest rates, households curtail their deposits to the financial sector. The result is that the banks are forced to rapidly sell their assets to maintain their balance sheets. The decreased demand for assets results in the stock market falling rapidly after its initial spike to below even the steady state level. While there was too much capital during the expansion, now there is too little. Return to optimality is slow due to the investment adjustment costs in the base model and investor subjectivity in the behavioral models. Though the output does not fall as sharply as it rose during impact, the duration of the recession exceeds the duration of the initial boom. The slump in GDP is associated with a prolonged decrease in labor supply and thereby in consumption, although this effect is much larger in the base model. In this manner, the results seem to agree with prior literature; a situation where over-leveraged agents are forced to rapidly deleverage due to economic conditions can lower aggregate demand, triggering a recession (see Eggertsson and Krugman, 2012). Note that the impulse responses match several facts of the mid-2000's U.S. economy: a few years of an economic boom corresponding with high increases in the leverage ratios of financial institutions followed by the crash of the 2008 Great Recession.

The size of the initial effect of the overconfidence shock on the economy is almost as high as the effect of a capital quality shock in the models with behavioral features. The models with behavioral features experience a significantly more volatile business cycle. The boom and bust are both accentuated as investors' subjective returns expectations overreact to the initial shock. While the IRFs look similar in both behavioral iterations, the propagation mechanisms are different. In K2023(a), the effects of the shock cognitively affect investors for several future quarters as they are anchored to prior returns when evaluating the future. In K2023(b), the initial expansion of net worth via increased stock returns causes investors' endogenous confidence in generating future returns to increase, thereby leading to over-investment which must be curtailed in the future. As both returns to capital and net worth are relatively quick to return to steady state, investors' subjective expectations also revert back quickly to rational expectations. Consequently, business cycles in K2023(b) and (c) are more volatile but less prolonged than baseline.

IV. Concluding Remarks

This paper includes subjective investor expectations in a medium-scale monetary DSGE model of the macroeconomy. Such subjective expectations are based on behavioral biases and/or heuristics such as anchoring and confidence. The empirical results confirm prior work in this field by demonstrating that in the presence of behavioral features, structural sources of persistence are less important. Additionally, the estimation shows that behavioral features are important in determining the model fit; anchoring and endogenous confidence are able to significantly improve the marginal likelihood.

Nevertheless, the models presented in this paper are stylized and offer paths for further nuance. Several other behavioral heuristics such as myopia, availability, representativeness among others, have not been included in this analysis. Future work can study these in isolation as well as in a model that incorporates several behavioral features to determine which is preferred by the data. Sensitivity tests are also required; using a variety of other expectation measures will help with model comparison and whether the results from this paper are sustained. Additionally, this paper does not indicate what effects, if any, such cognitive features should have on policy. If subjective expectations are important in explaining economic environments, they must also be important to social planners who aim to control these systems and mitigate its deleterious effects.

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APPENDIX A: BASE MODEL EQUILIBRIUM CONDITIONS

1. Marginal utility of consumption:

$$\varrho_t = (C_t - hC_{t-1})^{-\sigma} - \mathbb{E}_t[\beta h(C_{t+1} - hC_t)^{-\sigma}]$$

   where C_t is consumption, $0 < h < 1$ is the household's degree of habit formation, $0 < \beta < 1$ is the discount factor, and $\sigma > 0$ is the intertemporal elasticity of substitution.

2. Stochastic discount rate:

$$\Lambda_{t,t+1} = \frac{\varrho_{t+1}}{\varrho_t}$$

3. Euler equation:

$$1 = \mathbb{E}_t[\beta R_{t+1} \Lambda_{t,t+1}]$$

   where R_{t+1} is the gross real return on one-period bonds from t to $t + 1$.

4. Labor market equilibrium:

$$\chi L_t^{\varphi} = \varrho_t W_t$$

   where L_t is household's labor supply, W_t is the wage rate, $\chi > 0$ is the relative weight of labor to utility, and $\varphi > 0$ is the inverse Frisch elasticity of labor supply.

5. Growth rate of banks' assets:

$$x_{t,t+i} = \frac{Q_{t+i} S_{t+i}}{Q_t S_t}$$

   where $S_{j,t}$ is the amount of shares of non-financial firms that financial firms hold as assets in their balance sheets with Q_t being the price of each share.

6. Growth rate of banks' net worth:

$$z_{t,t+i} = \frac{N_{t+i}}{N_t}$$

   where N_t is banks' net worth or equity.

7. Value of banks' capital:

$$\nu_t = \mathbb{E}_t[(1 - \theta)\beta \Lambda_{t,t+1}(R_{k,t+1} - R_{t+1}) + \beta \Lambda_{t,t+1} \theta x_{t,t+1} \nu_{t+1}]$$

   where $R_{k,t+1}$ is the stochastic return on assets earned by the banker from t to $t+1$ and $0 < \theta < 1$ is bankers' survival rate.

8. Value of banks' net worth:

$$\eta_t = \mathbb{E}_t[(1 - \theta) + \beta \Lambda_{t,t+1} \theta z_{t,t+1} \eta_{t+1}]$$

9. Optimal leverage ratio:
$$\phi_t = \frac{\eta_t}{\lambda - \nu_t}$$
where $0 < \lambda < 1$ is the fraction of assets that may be diverted away by bankers.

10. Aggregate capital:
$$Q_t K_{t+1} = \phi_t N_t$$
where K_{t+1} is the capital acquired by intermediate goods producers. This capital is financed by funds obtained from the financial intermediaries.

11. Banks' aggregate net worth:
$$N_t = N_{e,t} + N_{n,t}$$
where $N_{e,t}$ and $N_{n,t}$ is the net worth of existing and new banks respectively.

12. Existing banks' net worth accumulation:
$$N_{e,t} = \theta z_{t,t-1} N_{t-1} \varepsilon_t^{N_e}$$
where $\varepsilon_t^{N_e}$ is an exogenous shock to existing banks' net worth.

13. New banks' net worth creation:
$$N_{n,t} = \omega Q_t \xi_t K_t$$
where ξ_t is the quality of capital and is governed by the AR(1) process: $\log \xi_t = \rho_\xi \log \xi_{t-1} + \varepsilon_t^{\xi}$. $0 < \omega < 1$ is the proportion of exiting banks' assets that is provided to new banks as "start up" funds.

14. Intermediate firms' production function:
$$Y_{m,t} = A_t (U_t \xi_t K_t)^\alpha L_t^{1-\alpha}$$
where A_t is the total factor productivity which is governed by the AR(1) process: $\log A_t = \rho_a \log A_{t-1} + \varepsilon_t^a$. U_t is the utilization rate of capital and $0 < \alpha < 1$ is the effective share of capital.

15. Optimal capacity utilization rate:
$$U_t^{1+\upsilon} = \frac{P_{m,t} \alpha Y_{m,t}}{b \xi_t K_t}$$
where $P_{m,t}$ is the price of intermediate firms' goods, υ is the elasticity of marginal depreciation with respect to the capital utilization rate, and b is the steady state value of the nominal marginal product of capital.

16. Depreciation rate:
$$\delta(U_t) = \delta_c + \frac{b}{1+\upsilon} U_t^{1+\upsilon}$$
where δ_c is set to maintain a steady state depreciation rate of 0.025.

17. Return to capital:
$$R_{k,t+1} = \frac{P_{m,t}\alpha \frac{Y_{m,t+1}}{K_{t+1}} + \xi_{t+1}(Q_{t+1} - \delta(U_{t+1}))}{Q_t}$$

18. Optimal investment decision:
$$Q_t = 1 + \frac{\eta_i}{2}\left(\frac{I_{n,t} - I_{n,t-1}}{I_{n,t-1} + I_{ss}}\right)^2 + \eta_i \left(\frac{I_{n,t} - I_{n,t-1}}{I_{n,t-1} + I_{ss}}\right)\left(\frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}}\right) - \mathbb{E}_t \beta \Lambda_{t,t+1} \eta_i \left(\frac{I_{n,t+1} - I_{n,t}}{I_{n,t} + I_{ss}}\right)\left(\frac{I_{n,t+1} + I_{ss}}{I_{n,t} + I_{ss}}\right)^2$$
where $I_{n,t}$ is the new capital created in the economy, I_{ss} is the steady state investment level, and η_i is the inverse elasticity of net investment to the price of capital.

19. Gross investment:
$$I_t = \delta(U_t)\xi_t K_t + I_{n,t}$$

20. Capital accumulation:
$$K_{t+1} = \xi_t K_t + I_{n,t}$$

21. Government expenditure:
$$G_t = G_{ss}\, g_t$$
where G_{ss} is steady state government spending and g_t is an exogenous disturbance that is modeled as the AR(1) process: $\log g_t = \rho_g \log g_{t-1} + \varepsilon_t^g$.

22. Aggregate resource constraint:
$$Y_t = C_t + G_t + I_t + \frac{\eta_i}{2}\left(\frac{I_{n,t} - I_{n,t-1}}{I_{n,t-1} + I_{ss}}\right)^2 (I_{n,t} + I_{ss})$$
where Y_t is the aggregate retail output in the economy.

23. Price dispersion:
$$D_t = \gamma D_{t-1} \pi_{t-1}^{-\gamma_p \epsilon} \pi_t^{\epsilon} + (1-\gamma)\left(\frac{1 - \gamma \pi_{t-1}^{\gamma_p(1-\gamma)} \pi_t^{\gamma-1}}{1-\gamma}\right)^{-\frac{\epsilon}{1-\gamma}}$$

where π_t is the economy's inflation rate from $t-1$ to t, $0 < \gamma < 1$ is the Calvo probability of firms having to keep prices fixed, $0 < \gamma_p < 1$ is the degree of price indexation, and ϵ is the elasticity of substitution across intermediate firms' products.

24. Retail output:

$$Y_t = \frac{Y_{m,t}}{D_t}$$

25. Pricing equation (1):

$$F_t = Y_t P_{m,t} + \mathbb{E}_t \beta \gamma \Lambda_{t,t+1} \pi_{t+1}^{\epsilon} \pi_t^{-\epsilon\gamma_p} F_{t+1}$$

26. Pricing equation (2):

$$Z_t = Y_t + \mathbb{E}_t \beta \gamma \Lambda_{t,t+1} \pi_{t+1}^{\epsilon-1} \pi_t^{\gamma_p(1-\epsilon)} Z_{t+1}$$

27. Optimal price choice:

$$\pi_t^* = \frac{\epsilon}{\epsilon - 1} \frac{F_t}{Z_t} \pi_t$$

28. Price index:

$$\pi_t^{1-\epsilon} = \gamma \pi_{t-1}^{\gamma_p(1-\epsilon)} + (1-\gamma) \pi_t^{*1-\epsilon}$$

29. Fisher equation:

$$i_t = R_{t+1} \mathbb{E}_t \pi_{t+1}$$

30. Taylor rule for interest rate:

$$i_t = i_{t-1}^{\rho_i} \left[\frac{1}{\beta} \pi_t^{\kappa_\pi} \left(P_{m,t} \frac{\epsilon}{\epsilon - 1} \right)^{\kappa_y} \right]^{1-\rho_i} \varepsilon_t^i$$

where $0 < \rho_i < 1$ is the interest rate smoothing parameter, κ_π is the inflation weight, κ_y is the output weight, and ε_t^i is an exogenous shock to monetary policy.

APPENDIX B: FULL POSTERIOR DISTRIBUTIONS

					Exo. Confidence			Anchoring ($T = 4$)			End. Confidence		
		Prior			K2023(a)			K2023(b)			K2023(c)		
Parameter	Description	Dist.	Mean	Dev.	Mean	10%	90%	Mean	10%	90%	Mean	10%	90%
h	Habit formation	Beta	0.70	0.10	0.8389	0.8088	0.8707	0.7915	0.7636	0.8291	0.8107	0.7812	0.8401
σ	IES	Gamma	1.50	1.00	0.5712	0.3407	0.8691	0.8369	0.6978	0.9583	0.7706	0.6738	0.8748
γ	Calvo factor	Beta	0.50	0.15	0.8172	0.7992	0.8364	0.9001	0.8923	0.9080	0.9009	0.8927	0.9090
γ_p	Price Indexation	Uniform	0.50	-	0.7720	0.5250	1.0000	0.0525	0.0000	0.1113	0.0578	0.0000	0.1224
η_i	Inv. Adjustment	Gamma	4.00	1.50	4.2032	4.0251	4.5325	0.1793	0.1539	0.2041	0.1765	0.1517	0.2007
κ_π	Taylor Rule	Normal	1.50	0.25	2.1320	1.9625	2.2676	1.8516	1.6237	2.0699	1.7866	1.5986	1.9984
κ_y	Taylor Rule	Normal	0.125	0.0625	0.3536	0.2883	0.4152	0.0547	0.0466	0.0628	0.0584	0.0509	0.0657
y^*	Trend	Normal	0.40	0.10	0.7426	0.7049	0.7755	0.9305	0.8979	0.9629	0.9360	0.9021	0.9705
π^*	Trend	Normal	0.60	0.10	0.5654	0.4629	0.6800	0.7707	0.6428	0.8902	0.7560	0.6371	0.9277
i^*	Trend	Normal	0.75	0.10	0.7043	0.5973	0.8489	0.7907	0.6828	0.8956	0.6725	0.5480	0.8036
β_0	Exp. Observation	Normal	0.00	0.30	0.0761	0.0574	0.0945	0.0747	0.0573	0.0940	0.0766	0.0599	0.0942
β_1	Exp. Observation	Normal	1.00	0.25	0.0167	0.0117	0.0220	0.0405	0.0195	0.0610	0.0393	0.0211	0.0561
ζ	Subjectivity	Uniform	0.50	-	-	-	-	0.1755	0.0000	0.4226	0.3144	0.0000	0.7424
ρ_1	Anchoring	Normal	0.90	0.25	-	-	-	0.6835	0.3164	0.9908	-	-	-
ρ_2	Anchoring	Normal	0.00	0.25	-	-	-	-0.2099	-0.5414	0.1721	-	-	-
ρ_3	Anchoring	Normal	0.00	0.25	-	-	-	0.1219	-0.1108	0.3913	-	-	-
ρ_4	Anchoring	-	-	-	-	-	-	0.4045	-0.5542	1.3358	-	-	-
ζ_c	Confidence	Gamma	2.00	1.50	-	-	-	-	-	-	1.5549	0.2356	2.9252

TABLE B1—POSTERIOR DISTRIBUTION OF STRUCTURAL PARAMETERS

Note: The table reports posterior distributions for the following models: under rational expectations [K2023(a)], with $T = 4$ quarters of anchoring [K2023(b)], and with endogenous confidence [K2023(c)]. All models include exogenous confidence shocks to investors' expectations formation process. All prior distributions are presented with means and standard deviations. Posterior means have been computed over two chains of 500,000 Metropolis-Hastings draws each and after a 40% burn-in. The data sample spans from Q1 1988 to Q4 2019.

Parameter	Description	Prior			Exo. Confidence K2023(a)			Anchoring ($T = 4$) K2023(b)			End. Confidence K2023(c)		
		Dist.	Mean	Dev.	Mean	10%	90%	Mean	10%	90%	Mean	10%	90%
Persistence													
ρ_a	Technology	Beta	0.50	0.20	0.3929	0.1454	0.6263	0.5243	0.4853	0.5617	0.5161	0.4726	0.5604
ρ_g	Govt. Spending	Beta	0.50	0.20	0.9489	0.9334	0.9644	0.9642	0.9528	0.9755	0.9651	0.9536	0.9767
ρ_i	Monetary Policy	Beta	0.50	0.20	0.3973	0.2254	0.5313	0.0932	0.0233	0.1577	0.1105	0.0218	0.1965
ρ_ξ	Capital Quality	Beta	0.50	0.20	0.2643	0.0766	0.4547	0.0123	0.0014	0.0227	0.0124	0.0015	0.0229
ρ_s	Ex. Confidence	Beta	0.50	0.20	0.3344	0.2639	0.3994	-	-	-	-	-	-
Deviation													
σ_a	Technology	Inv. Gamma	0.30	1.00	0.0394	0.0366	0.0425	0.0513	0.0426	0.0601	0.0533	0.0432	0.0631
σ_g	Govt. Spending	Inv. Gamma	0.30	1.00	0.0419	0.0377	0.0460	0.0443	0.0393	0.0490	0.0440	0.0392	0.0487
σ_i	Monetary Policy	Inv. Gamma	0.30	1.00	0.0371	0.0366	0.0378	0.0371	0.0366	0.0378	0.0371	0.0366	0.0378
σ_ξ	Capital Quality	Inv. Gamma	0.30	1.00	0.0377	0.0366	0.0392	0.0387	0.0366	0.0410	0.0390	0.0366	0.0415
σ_{Ne}	Net Worth	Inv. Gamma	0.30	1.00	0.2172	0.1716	0.2638	0.0514	0.0443	0.0587	0.0516	0.0442	0.0589
σ_s	Ex. Confidence	Inv. Gamma	0.30	1.00	2.0011	1.7211	2.1972	0.9318	0.8072	1.0532	0.9424	0.8179	1.0722
σ_{obs}	Measurement Error	Inv. Gamma	0.30	1.00	0.1112	0.0995	0.1225	0.1199	0.1070	0.1317	0.1191	0.1067	0.1305

TABLE B2—POSTERIOR DISTRIBUTION OF SHOCK PROCESSES

Note: The table reports posterior distributions for the following models: under rational expectations [K2023(a)], with $T = 4$ quarters of anchoring [K2023(b)], and with endogenous confidence [K2023(c)]. All models include exogenous confidence shocks to investors' expectations formation process. All prior distributions are presented with means and standard deviations. Posterior means have been computed over two chains of 500,000 Metropolis-Hastings draws each and after a 40% burn-in. The data sample spans from Q1 1988 to Q4 2019.